Numerical Simulation of Heat Transfer in Hydromagnetic NanofluidsFlowThrough Porous MediumOver a Stretching Sheet With Thermal Radiation

Kapil Kumar
Deptt. of Mathematics
Meerut College, Meerut
Email:

Dr. Bhupander Singh
Deptt. of Mathematics
Associate Professor
Meerut College, Meerut

Dr. S.K. Sharma
Deptt. of Physics
Associate Professor
NAS College, Meerut

Reference to this paper should be made as follows:

Kapil Kumar
Dr. Bhupander Singh
Dr.. S.K. Sharma

" *Numerical Simulation of Heat Transfer in Hydromagnetic NanofluidsFlowThrough***",**

Voyager: Vol. X, 2019
pp.20-35

Abstract

In this paper an investigation of heat transfer in nanofluid flow over a stretching sheet in porous medium is worked out. Similarity equation are solved numerically by BVP4C package MATLAB and the influence of various parameters like magneto porous parameter, thermal radiation parameter and Prandtl number on simulated velocity profile and temperature profile have been observed and the resultsso obtained are explainedgraphically

Introduction

Nanofluids studies have attracted on considerable attention of researchers due to many industrial and technological application that includes both metal and polymer sheets. Heat transfer enhancement is an interesting phenomenon of nanofluids that attracts various researchers. To improve the thermal properties of nanofluids introduce by the scientist and researchers. Crane [1] was the first who gave a closed analytical form exact solution for steady boundary layer flow of a viscous fluid past a stretching sheet. Choi et al. [2] reported that on adding nanoparticles in the base fluid, the thermal conductivity of liquids is enhanced to double. The effective thermal conductive are explained and compared between experimental findings and theoretical predictions of his work. Vleggaar [3], Gupta and Gupta [4] have discussed a continuous linearly stretching surface with constant surface temperature while Ramana Murty and Soundalgekar [5] examined a constant surface velocity case with power law temperature variation. Dutta and Gupta [6] presented the coupled heat transfer problem over stretching sheet in extension zone in the fluid and made the conclusion that for a fixed Prandtl number, the surface temperature decreases with an increase in the stretching speed. Numerical study for the boundary layer flow of a nanofluid past a stretching sheet has been presented by Khan and Pop [7]. They also included in his work the Brownian motion and thermophoresis effects. Cortell [8] studied the flow and heat transfer of an incompressible homogeneous second grade fluid over a stretching sheet.In recent years, most of the researchers are showing their interest on studies of nanofluid over stretchingsheet in porous medium.

Performance of porous media and nanofluid enhanced the thermal properties of nanofluid, such as convective heat transfer coefficients and higher thermal conductivity on comparing with the base material [9]. In porous medium, the nanofluid flow have numerous applications in the area of thermal engineering, like as, vehicle cooling, transformer cooling, cooling equipment, filtration and help to improve the thermal properties of oil and lubricants [10]. The nanofluid flow and heat transfer in porous medium have also studied by the researchers [11-18].

To best of our knowledge, Kameswaran et al. [19] studied the hydromagnetic nanofluid flow over a stretching sheet with chemical reaction effects and viscous dissipation. They investigated the convective heat and mass transfer in nanofluid flow over a stretching sheet. Dulal and Gopinath [20] analysed effects of thermal radiation on mixed convection stagnation point flow of nanofluids towards apermeable stretching/shrinking sheet embedded in a porous medium with chemical reaction. Nasir et al. [21]presented the MHD flow and heat transfer of couple stress fluid past an oscillatory stretching sheet in the presence of heat source and sink embedded in porous medium. Shit et al. [22] have expounded the entropy generation on unsteady magnetohydrodynamic flow and convective transfer of exponentially stretching surface in porous medium saturated by nanofluids in the presence of thermal radiation. Mirzaaghaian and Gajni [24] studied the micropolar fluid flow and heat transfer over a channel with permeable wall by differential transformation method. They found that differential transformation method is the suitable method to verify the accuracy and validity in the solution of the application. In the presence of magnetic field, unsteady two phases nanofluid flow and heat transfer between moving parallel plates by differential transformation method was examined by Usman M. et. al. [25]. They obtain the numerical results of various values of parameters such as the squeeze number, Hartmann number and Eckert number. In the presence of variable magnetic field, the problem of the behaviour of nanofluid hydrothermal was investigated analytically by Sheikholeslami and Ganji [26]. They showed that when Hartmann number and squeeze number increases, the skin friction coefficient increases but skin friction coefficient decreases when increases the nanofluid volume friction. The Nusselt number decreases with increases of squeeze number but it increases with augment of nanoparticle volume fraction, Hartmann number. Sepasgozar et al. [27] used differential transformation method to find the solution of momentum and heat transfer equations of non-Newtonian fluid flow in an axisymmetric channel through porous wall. They concluded that friction force and Nusselt number increases with increase in Reynolds number and the temperature value between two plates decreases as a result of power law index increases.

Mathematical formulation

Consider a two-dimensional incompressible magne tohydrodynamic nanofluid flow over a stretching sheet through a porous medium. In a rectangular coordinate system $(\bar{\mathbf{x}}, \bar{\mathbf{y}})$, the sheet is assumed to coincide with $\bar{\mathbf{y}} = 0$ and flow takes place in semi-infinite porous space ($\bar{\mathbf{y}} > 0$) of constant permeability (see fig. 1). The flow is subjected to a constant magnetic field of strength $\mathbf{B_0}$ applied normal to the stretching surface. It is assumed that the sheet is stretched with velocity $\mathbf{u_w = b\bar{x}}$, where b > 0, representing the stretching rate. The sheet is maintained at a uniform temperature $\mathbf{T_w > T_\infty}$ where $\mathbf{T_\infty}$ is the temperature of the ambient fluid.

The problem is to obtain the velocity and temperature fields inside the fluid satisfying the boundary conditions at the wall and far from wall. To this end, the governing equations under the boundary layer approximation can be composed as

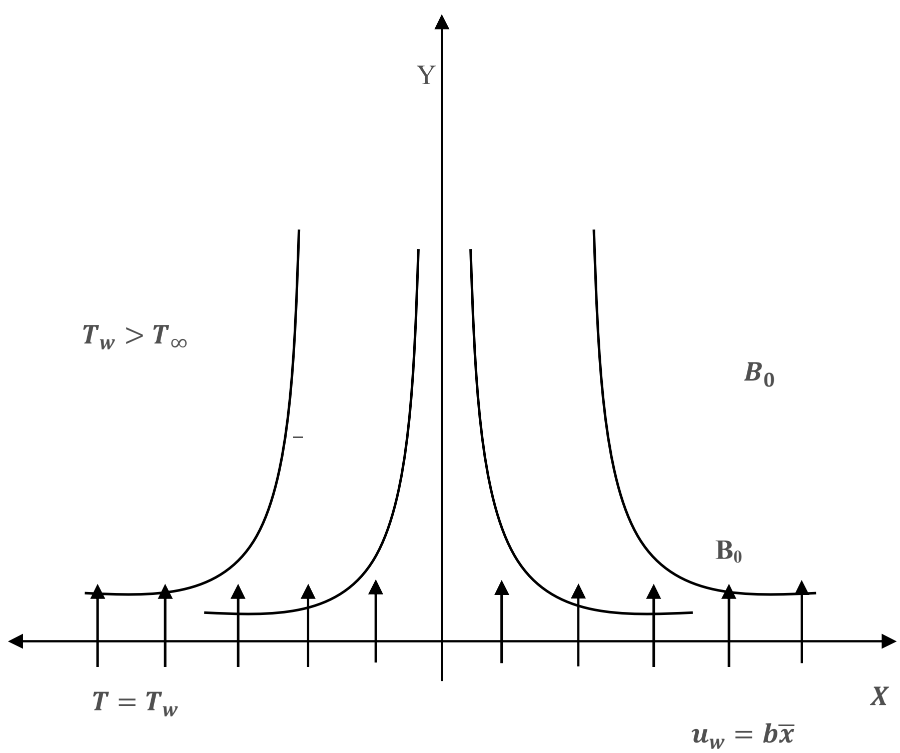

Figure 1. Geometry of problem.

$$\frac{\partial u}{\partial \bar{x}} + \frac{\partial v}{\partial \bar{y}} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial \bar{x}} + v\frac{\partial v}{\partial \bar{y}} = \nu\frac{\partial^2 u}{\partial \bar{y}} - \frac{\sigma {B_0}^2}{\rho}u - \frac{\nu\varphi}{k}u \quad (2)$$

$$u\frac{\partial T}{\partial \bar{x}} + v\frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial \bar{y}} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial \bar{y}} \quad (3)$$

where $\mathbf{u}$ is the velocity component along $\bar{\mathbf{x}}$ direction and $\mathbf{v}$ is the velocity component along $\bar{\mathbf{y}}$ direction, $\mathbf{T}$ is the temperature of nanofluid, ν is the kinematic viscosity, $\boldsymbol{\sigma}$ is electric conductivity, $\boldsymbol{\rho}$ is the density, $\mathbf{B_0}$ is the constant of strength magnetic field. The appropriate boundary conditions are given by

$$u_w = b\bar{x},\ v = 0,\ \frac{\partial^2 u}{\partial y^2} = 0, T = T_w \quad at \quad \bar{y} = 0 \quad (4)$$

$$u = 0,\ \frac{\partial u}{\partial \bar{y}} = 0,\quad T = T_\infty \quad at \quad \bar{y} \to \infty \quad (5)$$

The Radiative heat flux under the Rosseland approximation for Radiation [23] is given by

$$q_r = -\frac{4\,\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (6)$$

where σ^* is the Stephan Boltzmann constant andk^* is the Rosseland mean absorptioncoefficient.It is further assumed that the temperature diffusion with in the flow are sufficiently small, we may expand the term T^4 due to radiation as a linear function of the temperature in Taylor series aboutT_∞ and can be approximated after neglecting the higher order terms as

$$T^4 \approx 4{T_\infty}^3 T - 3{T_\infty}^4. \quad (7)$$

Using Equations (6) and (7), we obtain

$$\frac{\partial q_r}{\partial \bar{y}} = -\frac{16\,\sigma^*\,{T_\infty}^3}{3k^*}\frac{\partial^2 T}{\partial \bar{y}^2}. \quad (8)$$

Using the following similarity transformations

$$\eta = \sqrt{\frac{b}{\nu}}\,\bar{y},\ u = bxf'(\eta),\quad v = -\sqrt{b\,\nu}\,f(\eta), \quad (9)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \quad (10)$$

the governing nonlinear partial differential equations (3) and (4) are transformed into a system of ordinary differential equations in non-dimension form by

$$f''' - \beta f' - f'^2 + ff'' = 0, \quad (11)$$

$$(1 + Nr)\theta'' + Pr\, f\theta' = 0 \quad (12)$$

Subject to the following boundary conditions

$$f(0) = 0,\quad f'(0) = 1,\quad \theta(0) = 1 \quad at\ \eta = 0 \quad (13)$$

$$f''(\infty) = 0,\quad f'(\infty) = 0,\ \theta(\infty)$$

$= \mathbf{0} \quad at\ \eta = \infty$ (14)

In above equations (11) and (12), the non-dimensional parameters are, $\boldsymbol{\beta}$, $\mathbf{Nr}$ **and** $\mathbf{Pr}$.

Where $\boldsymbol{\beta} = \frac{\sigma B_0{}^2}{b\rho} + \frac{\xi\varphi}{kb}$ is the magneto porous parameter, $\boldsymbol{Nr} = \frac{16\sigma^*}{3kk^*}\boldsymbol{T}_\infty{}^3$ is the thermal radiation parameter and $\boldsymbol{Pr} = \frac{\nu}{k}\boldsymbol{\rho c_p}$ is the Prandtl number.

The mathematical expressions for various physical quantities of interest like skin friction coefficient $\mathbf{C_f}$, local Nusselt number $\mathbf{N_u}$ are defined as

$$\boldsymbol{C_f} = \frac{\boldsymbol{\tau_w}}{\boldsymbol{\rho u_w}^2}, \quad \boldsymbol{Nu_x} = \frac{\bar{x}q_w}{k(T_w - T_\infty)}, \tag{15}$$

Where

$$\boldsymbol{\tau_w} = \boldsymbol{\mu}\left(\frac{\partial \boldsymbol{u}}{\partial \bar{\boldsymbol{y}}}\right)_{\bar{y}=0}, \quad \boldsymbol{q_w} = -\boldsymbol{k}\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}, \tag{16}$$

Utilizing Equations (9) and (10), we obtain

$$\boldsymbol{Re_x}^{\frac{1}{2}}\boldsymbol{C_f} = \boldsymbol{f''(0)}, \quad \boldsymbol{Re_x}^{-1/2}\boldsymbol{Nu_x} = -\boldsymbol{\theta'(0)}, \tag{17}$$

Where $\boldsymbol{Re_x} = \frac{u_w\bar{x}}{\nu}$ is the local Reynolds number.

Basics of differential transformation method [27].

Basic definitions and operations of differential transformation are presented as follows. Differential transformationof the $\mathbf{k^{th}}$derivative of a function $\mathbf{f}(\eta)$ are defined as follows:

$$\boldsymbol{F(k)} = \frac{1}{k!}\left[\frac{d^k f(\eta)}{d\eta^k}\right]_{\eta=\eta_0} \tag{18}$$

Where $\mathbf{F(k)}$ is called the T-function of $\mathbf{f}(\eta)$at $\eta = \eta_0$ in k-domain. The differential inverse transformation of $\mathbf{F(k)}$ is defined as:

$$\boldsymbol{f}(\eta) = \sum_{k=0}^{\infty} \boldsymbol{F(k)}(\eta - \eta_0)^{k} \tag{19}$$

By combining Equation (18) and Equation (19), $\mathbf{f}(\eta)$can be obtained

$$\boldsymbol{f}(\eta) = \sum_{k=0}^{\infty}\left[\frac{d^k f(\eta)}{d\eta^k}\right]_{\eta=\eta_0} \frac{(\eta-\eta_0)^k}{k!} \tag{20}$$

In practice, to achieve a finite one, this infinite series truncated as follows:

$$\boldsymbol{f}(\eta) = \sum_{k=0}^{N}\left[\frac{d^k f(\eta)}{d\eta^k}\right]_{\eta=\eta_0} \frac{(\eta-\eta_0)^k}{k} \tag{21}$$

Where $\boldsymbol{f}(\eta) = \sum_{N+1}^{\infty}\left[\frac{d^k f(\eta)}{d\eta^k}\right]_{\eta=\eta_0}$ is negligibly small. Generally, the value of N is decided by convergence of the series coefficient.

Solution of the problem by DTM:

Taking the one-dimensional differential transform of each differential equation by using table 1, we obtain the following results:

$\boldsymbol{(k+1)(k+2)(k+3)F[k+3]} - \boldsymbol{\beta(k+1)F[k+1]} - \sum_{l=0}^{k}\boldsymbol{(l+1)F[l+1](k-l+1)F[k-l+1]} +$

$$\sum_{l=0}^{k} F[l](k-l+1)(k-l+2)F[k-l+2] = 0 \quad (22)$$

$$(1+\mathrm{Nr})(k+1)(k+2)\Theta[k+2] + \mathrm{Pr}\sum_{l=0}^{l=k}(l+1)F[l+1](k-l+1)\Theta[k-l+1] = 0 \quad (23)$$

With boundary conditions.

$$F[0] = 0,\ \ F[1] = 0,\ \ F[2] = \alpha_1,\ \ \Theta[0] = 1,\ \ \Theta[1] = \alpha_2, \quad (24)$$

To find an analytic solution by differential transformation method is not possible which satisfies the boundary condition at infinity. So, we apply the Pade approximation the value of coefficients α_1 and α_2 can be determined. Finally, by substituting the obtained coefficients α_1 and $\alpha_2$into the Equations (22) and (23),the expressions of $\mathbf{f}(\eta)$ and $\Theta(\eta)$ can be obtained in case of$\boldsymbol{\beta} = \mathbf{0.5}$, $\mathbf{Nr} = \mathbf{0.1}$ and$\mathbf{Pr} = \mathbf{0.6}$ as

$$\mathbf{f}(\eta) = \eta + 0.63809867\,\eta^2 + 0.25\,\eta^3 + 0.079762333\,\eta^4 + 0.01982233\,\eta^5 + 3.988116688\text{x}10^{-3}\eta^6 + \ldots\ldots.. (25)$$

$$\Theta(\eta) = 1 + 0.122371\,\eta - 0.3337390909\,\eta^2 + 0.008129309915\,\eta^3 + 8.087735188\text{x}10^{-4}\eta^4 + 3.502133455\text{x}10^{-3}\eta^5 + \ldots\ldots\ldots.. \quad (26)$$

Results and discussions

In this section, a brief study of the effect of various parameters like, Magneto porous parameter, Prandtl number, Thermal Radiation on hydromagnetic nanofluid flow over stretching sheet in porous medium is to be discussed.The influence of skin friction coefficient and Nusselt number for different values of parameter are also discussed. The behaviour of velocity profile and temperature profile can be found with the variation of several parameters of the nanofluid. Figure 2. shows that when increases the Magneto porous parameter, the velocity profile decreased with the fix value of Thermal Radiation parameter and Prandtl number.Figure 3 is the plot of temperature profile for various values of magneto porous parameter. It is clearly observed from this figure that as the magneto porous parameter$\boldsymbol{\beta}$ increases, the temperature profile increases with the fix value of thermal radiation parameter and the Prandtl number. The effect of thermal Radiation parameter on temperature profile is illustrated in figure 4. It shows that temperature profileincreaseson increasingthermal Radiation parameter with the fix value of Magneto porous parameter and Prandtl number. Figure 5 shows that when the Prandtl numberincreases, the temperature profile decreases with the fix value of Thermal Radiation parameter and Magneto porous parameter.

Figure 2. Effect of magneto porous parameter β on velocity profile.

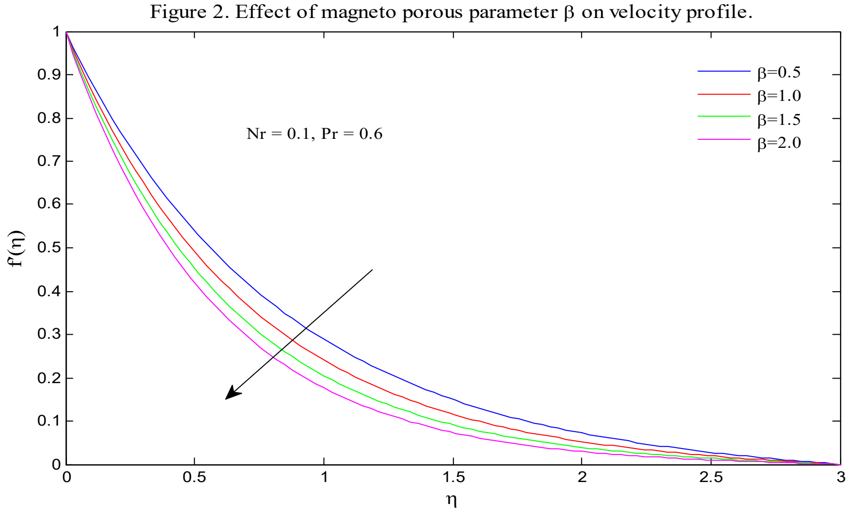

Figure 3. Effect of magneto porous parameter β on temperature profile.

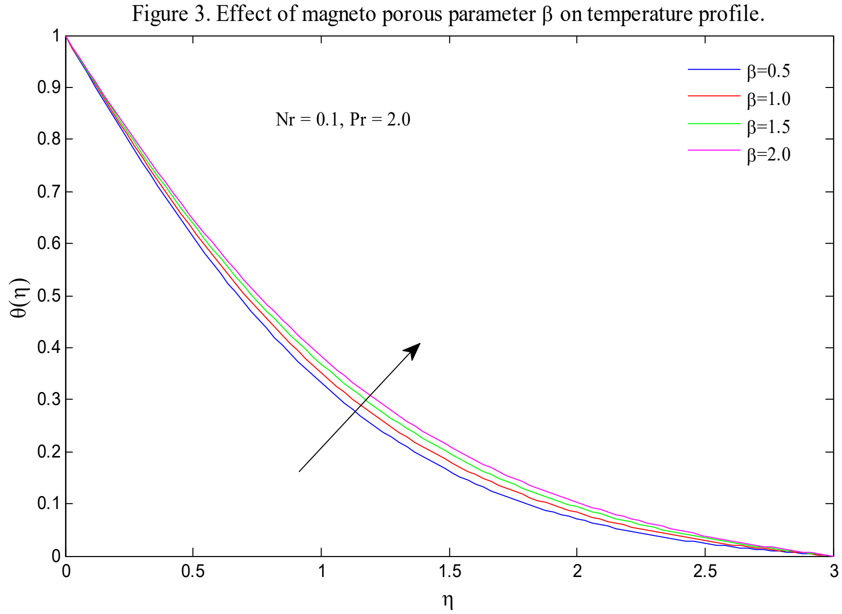

Figure 4. Effect of thermal radiation parameter Nr on temperature profile.

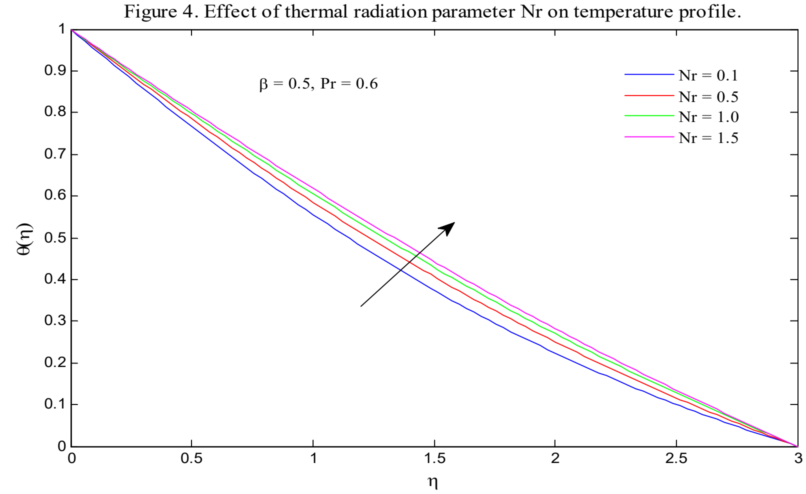

Figure 5. Effect of Prandtl number Pr on temperature profile.

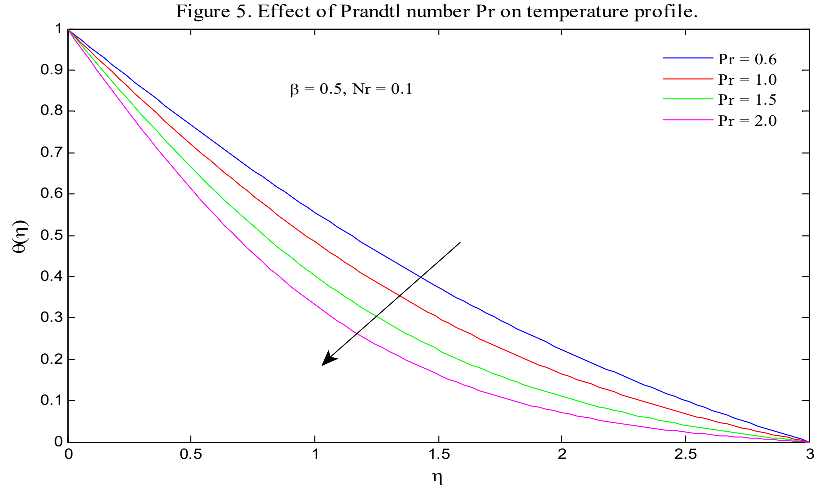

Table 1.

$\boldsymbol{\beta}$	Nr	Pr	$-f''(0)$	$-\theta'(0)$
0.5	0.1	0.6	1.2295	0.4750
0.6	0.1	0.6	1.2688	0.4727
0.7	0.1	0.6	0.3070	0.4704
0.5	0.1	0.6	1.2295	0.4750
0.5	0.2	0.6	1.2295	0.4626
0.5	0.3	0.6	1.2295	0.4522
0.5	0.1	0.6	1.2295	0.4750
0.5	0.1	0.7	1.2295	0.4999
0.5	0.1	0.8	1.2295	0.5250

The skin friction coefficient $-f''(0)$and the wall temperature gradient $-\theta'(0)$ for different values of magneto porous parameter $\boldsymbol{\beta}$, Thermal radiation parameter Nr and Prandtl number Pr.

The velocity profile decreases with increment of magneto porous The temperature profile increases with increment of magneto porous parameter $\boldsymbol{\beta}$for the fix values of Prandtl number and thermal radiation parameter. The temperature profile increases with increment of thermal radiation parameter Nr for the fix values of magneto porous parameter and Prandtl number. The temperature profile decreases with increment of Prandtl number for the fix

Conclusion

In this analysis we have presented heat transfer of non-Newtonian nanofluid flow over a stretching sheet in the presence of thermal radiation in porous medium. The conclusion of the results are as follows:

parameter $\boldsymbol{\beta}$for the fix values of Prandtl number and thermal radiation parameter.

values of magneto porous parameter and thermal radiation parameter.

The skin friction coefficient increases on increasing magneto porous parameter for the fix values of thermal radiation parameter and Prandtl number.

The wall temperature gradient decreases onincreasingmagneto porous parameter for the fix value of thermal radiation parameter The wall temperature gradient also decreases on increasing thermal radiation parameter for the fix values of magneto porous parameter and Prandtl number.

References

1. Crane,L.J., Angew.Z. (1970). Math. Mech. 21, **645–647**.
2. Choi, S.U.S., Zhang, Z.G., Yu, W., Lockwood, F.E.& Grulke, E.A. (2001). Anomalously thermal conductivity enhancement in nano tube suspensions. *Appl. Phys. Lett.* 79, **2252–2254**
3. Vleggaar, J. (1977). Laminar Boundary-Layer Behavior on Continuous, Accelerating Surfaces.*Chemical Engineering Science*, 32, **1517-1525**.
4. Gupta, P. S., &Gupta, A. S. (1977). Heat and Mass Transfer on a Stretching Sheet with Suction or Blowing.*Canadian Journal of Chemical Engineering*, 55(6), **744-746.**
5. Soundalgekar, V. M., &Ramana Murty, T. V. (1980). Heat Transfer Past a Continuous Moving Plate with Variable Temperature. *Wärme - und Stoffübertragung* , 14**, 91-93**.
6. Dutta, B. K., & Gupta, A. S. (1987). Cooling of a stretching sheet in a viscous flow. *Industrial & Engineering Chemistry Research*, 26(2**), 333–336**.
7. Khan, W. A., &Pop, I. (2010). Boundary-layer flow of a nanofluid past a stretching sheet.*Int. J. Heat Mass Transfer*, 53**, 2477 – 2483**.
8. Cortell, R. (2006). A note on flow and heat transfer of a viscoelastic liquid over a stretching sheet.*Int. J. Non-Linear Mech.*, 41, **78-85**.
9. Abed Mahdi, R., Mohammed, H. A., &Munisamy, K. M. (2013). Improvement of convection heat transfer by using porous media and nanofluid: Review.*Int. J. Sci. Res. (IJSR)*, India Online ISSN: **2319–7064**.
10. James, M.,Mureithi, E.W.&Kuznetsov, D. (2014). Natural convection flow past an impermeable vertical plate embedded in nanofluid saturated porous medium with temperature dependent viscosity.*Asian J. Math. Applic*., Article ID: ama0165, 17 pages.
11. Uddin, Z.&Harmand, S. (2013). Natural convection heat transfer of nanofluids along a vertical plate embedded in porous medium.*Nanoscale Res. Lett*. 8**, 64**.
12. Chamkha, A. J., Aly, A. M., & Mansour, M. A. (2010). Similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/injection and chemical reaction effects. *Chemical Engineering Communications*, 197(6**), 846–858.**

13. Das, K.(2014).Flow and heat transfer characteristics of nanofluids in a rotating frame. *Alex. Eng. J*., 53**, 757-766.**
14. Barik, R., Dash, G.C.,&Kar, M.(2014).Unsteady free convective MHD flow and mass transfer through porous medium in a rotating system with fluctuating heat source/sink and chemical reaction.*J. Appl. Anal. Comput*., 4**, 231-244**.
15. Zhang, C., Zheng, L., Zhang, X.,&Chen, G.(2015), MHD flow and radiation heat transfer of nanofluids in porous media with variable surface heat flux and chemical reaction.*Appl. Math. Modell.,*39, **165–181.**
16. Yadav, D., Lee, J. & Cho, H.H.(2015). Brinkman convection induced by purely internal heating in a rotating porous medium layer saturated by a nanofluid.,*Powder Tech*., 286, **592-601**.
17. Ghalambaz, M., Behseresht, A.,Behseresht, J., &Chamkha, A.(2015). Effects of nanoparticles diameter and concentration on natural convection of the Al2O3–water nanofluids considering variable thermal conductivity around a vertical cone in porous media.*Adv. Powder Tech.,* 26**, 224-235.**
18. Ramana Reddy, J.V., Sugunamma, V., Sandeep, N., &Sulochana, C.(2016). Influence of chemical reaction, radiation and rotation on MHD nanofluid flow past a permeable flat plate in porous medium.*J. Nigerian Math. Soc.,* 35**, 48–65.**
19. Kameswaran, P. K., Narayana, M., Sibanda, P., & Murthy, P. V. S. N. (2012). Hydromagnetic nanofluid flow due to a stretching or shrinking sheet with viscous dissipation and chemical reaction effects. *International Journal of Heat and Mass Transfer*, 55(25-26), **7587–7595.**
20. Pal, D., & Mandal, G. (2014). Influence of thermal radiation on mixed convection heat and mass transfer stagnation-point flow in nanofluids over stretching/shrinking sheet in a porous medium with chemical reaction. *Nuclear Engineering and Design*, 273, **644–652.**
21. Ali, N., Ullah Khan, S., Sajid, M., & Abbas, Z. (2016). MHD flow and heat transfer of couple stress fluid over an oscillatory stretching sheet with heat source/sink in porous medium. *Alexandria Engineering Journal,* 55(2**), 915–924.**
22. Shit, G. C., Haldar, R., & Mandal, S. (2017). Entropy generation on MHD flow and convective heat transfer in a porous medium of exponentially stretching surface saturated by nanofluids. *Advanced Powder Technology*, 28(6), **1519–1530**.
23. Rosseland, S.(1931). Springer-Verlag, *Berlin,* **41–44**.
24. Mirzaaghaian, A., & Ganji, D. D. (2016). Application of differential transformation method in micropolar fluid flow and heat transfer through permeable walls. *Alexandria Engineering Journal*, 55(3), **2183–2191.**

25. Usman, M., Hamid, M., Khan, U., Mohyud Din, S. T., Iqbal, M. A., & Wang, W. (2017). Differential transform method for unsteady nanofluid flow and heat transfer. *Alexandria Engineering Journal.*
26. Sheikholeslami, M., & Ganji, D. D. (2015). Nanofluid flow and heat transfer between parallel plates considering Brownian motion using DTM. *Computer Methods in Applied Mechanics and Engineering*, 283**, 651–663.**
27. Sepasgozar, S., Faraji, M., & Valipour, P. (2017). Application of differential transformation method (DTM) for heat and mass transfer in a porous channel. *Propulsion and Power Research*, 6(1**), 41–48**.